

# Study on the properties of the coupling constants of $J/\psi \rightarrow VP$ decays

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Basing on the branching fractions of  $J/\psi \rightarrow VP$  from different experiments, we investigate on the properties of the coupling constants of  $J/\psi \rightarrow VP$  decays with a model-dependent approach. We find that the octet coupling constant,  $g_8$ , of strong interaction is about twice larger than that of the singlet coupling constant  $g_1$ ; the electromagnetic breaking parameters  $g_E^i$  are larger than the mass breaking parameters  $g_M^i$ , moreover, the three parameters of electromagnetic effect are about equal, but the three parameters of mass effect are obviously different and their uncertainties are also large; and the phase angle between strong and electromagnetic interaction is in the range of  $70^\circ \sim 80^\circ$ . It deepens our understanding of the coupling constant of  $J/\psi \rightarrow VP$  decays.

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## I. INTRODUCTION

The  $J/\psi$  decays play an important role in the understanding of low-energy hadron dynamics. A particular advantage of these decays is that the initial state is a very good approximation of a flavor- $SU(3)$  singlet. In order to estimate that  $SU(3)$  is an approximate symmetry of low-energy hadron interactions, we need study in detail of the information on the final-state particles of  $J/\psi$  decay.

The implication of  $SU(3)$  symmetry to  $J/\psi$  decays into mesons has been studied [1].  $J/\psi$  can decay into a vector and a pseudoscalar induced by three-gluon annihilation and electromagnetic processes. However, these branching ratios are only of the order  $10^{-3}$  since the hadronic decays of  $J/\psi$  are suppressed by the Okubo-Zweig-Iizuka (OZI) rule [2].

Based on the  $SU(3)$  symmetry, three quarks may make up an octet and one singlet. Because the mass of  $s$  quark is larger than those of  $u$  and  $d$  quarks and the mixing in the mesons exists, it will cause  $SU(3)$  breaking and further create a physical nonet. The octet and singlet are the eigenstates of  $SU(3)$  group, however, what we can observe in experiment are not  $SU(3)$  eigenstates themselves, but their mixing. For example, for the pseudoscalar mesons, if pseudoscalar glueballs and radially excited states are ignored and only quark states are considered, then the physical states  $\eta$  and  $\eta'$  are related to

$\eta^8$  and  $\eta^0$ , via the usual mixing formulas

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_P & -\sin\theta_P \\ \sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta^8 \\ \eta^0 \end{pmatrix}, \quad (1)$$

where  $\theta_P$  is the mixing angle between  $\eta^8$  and  $\eta^0$ . Similarly, for the vector mesons  $\phi$  and  $\omega$ , we have

$$\begin{pmatrix} \phi \\ \omega \end{pmatrix} = \begin{pmatrix} \cos\theta_V & -\sin\theta_V \\ \sin\theta_V & \cos\theta_V \end{pmatrix} \begin{pmatrix} \omega^8 \\ \omega^0 \end{pmatrix}, \quad (2)$$

where  $\theta_V$  is the mixing angle between  $\omega^8$  and  $\omega^0$ .

For vector mesons, the mixing between  $\omega^8$  and  $\omega^0$  is basically an idea mixing,  $\sin\theta_V = \sqrt{1/3}$ ,  $\cos\theta_V = \sqrt{2/3}$ , i. e.  $\theta_V \approx 35.3^\circ$ , so in this case we have

$$\omega = \sqrt{\frac{1}{2}}|u\bar{u} + d\bar{d}\rangle, \quad \phi = |s\bar{s}\rangle. \quad (3)$$

However, for pseudoscalar mesons, it always brings people great interesting to discuss the mixing between  $\eta^8$  and  $\eta^0$  [3]-[10].

Combining the mixings in vector mesons and pseudoscalar mesons and considering the various mass effects and electromagnetic effects, in this paper we shall analyse the properties of the coupling constants of  $J/\psi \rightarrow VP$  decay, which is very important to comprehend the breaking of  $SU(3)$  flavor symmetry.

## II. PHENOMENOLOGICAL STUDY ON THE DECAYS $J/\psi \rightarrow VP$

### A. Effective Lagrangian of $J/\psi \rightarrow VP$

For the two-body decays  $J/\psi \rightarrow H_1 H_2$ , where  $H_1$  and  $H_2$  denote mesons, the most general interaction term in

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an  $SU(3)$  invariant is [2]

$$L_{\text{int}} = \psi(g_8 O_1^a O_2^a + g_1 S_1 S_2), \quad (4)$$

where  $O$  and  $S$  denote an octet and a singlet, respectively, and a sum over  $a = 1, 2, \dots, 8$  is implied.

The  $SU(3)$  symmetry in the nonet of pseudoscalar meson is not strict, two types of  $SU(3)$  breaking should be considered. The first one is induced by the different mass of quark, and the other one is the different charge of quark.

In our theoretical calculation,  $m_u = m_d$  is usually assumed, but  $m_s \neq m_d$ , this difference of mass will cause the breaking effect on  $SU(3)$  symmetry

$$m_d(\bar{u}u + \bar{d}d) + m_s \bar{s}s = m_0 \bar{q}q + \frac{1}{\sqrt{3}}(m_d - m_s) \bar{q} \lambda_8 q, \quad (5)$$

where  $q = (u, d, s)$ ,  $m_0 = \frac{1}{3}(2m_d + m_s)$  is the average quark mass, and  $\lambda_8$  is the eighth one of Gell-Mann matrices. The last term in Eq. (5) is the mass effects of violating  $SU(3)$  invariance. We need introduce a new spurion  $M^a = \delta^{a8}$  to describe this  $SU(3)$  breaking term.

The electromagnetic effects violate  $SU(3)$  symmetry since the photon coupling to quarks is proportional to the electric charge

$$\frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d - \frac{1}{3} \bar{s} \gamma_\mu s = \frac{1}{2} \bar{q} \gamma_\mu (\lambda_3 + \frac{1}{\sqrt{3}} \lambda_8) q, \quad (6)$$

where  $\lambda_3$  and  $\lambda_8$  denote the third and eighth of Gell-Mann matrices, respectively. It follows from Eq. (6) that the electromagnetic breaking can be simulated by a spurion  $E = \delta^{a3} + \frac{1}{\sqrt{3}} \delta^{a8}$ .

After the above two effects of  $SU(3)$  breaking are considered, the effective Lagrangian of  $J/\psi \rightarrow H_1 H_2$  process can be written [2]

$$\begin{aligned} \mathcal{L}_{\text{eff}} = \psi \Big\{ & g_8 O_1^a O_2^a + g_1 S_1 S_2 + g_S d_{abc} O_1^a O_2^b O_3^c \\ & + g_A f_{abc} O_1^a O_2^b O_3^c + \sqrt{\frac{2}{3}} \left[ C_{123} O_1^a O_2^a S_3 \right. \\ & \left. + C_{132} O_1^a O_3^a S_2 + C_{231} O_2^a O_3^a S_1 + f S_1 S_2 S_3 \right] \Big\}. \quad (7) \end{aligned}$$

If the final states of  $J/\psi \rightarrow H_1 H_2$  decays are a vector and a pseudoscalar, the expression of the above effective Lagrangian can be further written as

$$\begin{aligned} \mathcal{L}_{\text{eff}} = \psi \Big\{ & g_8 P_1^a V_2^a + g_1 P_0 V_0 + g_M^{88} d_{abc} O_1^a O_2^b M_3^c \\ & + \sqrt{\frac{2}{3}} \left[ g_M^{81} O_1^a M_3^a S_2 + g_M^{18} O_2^a M_3^a S_1 \right] + g_E^{88} d_{abc} O_1^a \\ & \times O_2^b E_3^c + \sqrt{\frac{2}{3}} \left[ g_E^{81} O_1^a E_3^a S_2 + g_E^{18} O_2^a E_3^a S_1 \right] \Big\}, \quad (8) \end{aligned}$$

in which  $g_8$  and  $g_1$  denote the coupling constants of octet and singlet,  $g_M^i$  and  $g_E^i$  are the coupling constants of the

mass breaking term and electromagnetic breaking term, respectively,  $f_{abc}$  and  $d_{abc}$  coefficients are the antisymmetrical and symmetrical structure constants of  $SU(3)$  group.

## B. Decay amplitude and width of $J/\psi \rightarrow VP$ decays

The physical particles in correspondence with the pseudoscalar and vector mesons are

$$\begin{aligned} P_1 &= \frac{1}{\sqrt{2}}(\pi^+ + \pi^-), \\ P_2 &= \frac{i}{\sqrt{2}}(\pi^+ - \pi^-), \\ P_3 &= \pi^0, \\ P_4 &= \frac{1}{\sqrt{2}}(K^+ + K^-), \\ P_5 &= \frac{i}{\sqrt{2}}(K^+ - K^-), \\ P_6 &= \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0) = K_S^0, \\ P_7 &= \frac{i}{\sqrt{2}}(K^0 - \bar{K}^0) = iK_L^0, \\ P_8 &= \eta_8^0 = \eta \cos \theta_P + \eta' \sin \theta_P, \\ P_0 &= \eta_1^0 = -\eta \sin \theta_P + \eta' \cos \theta_P, \end{aligned} \quad (9)$$

and

$$\begin{aligned} V_1 &= \frac{1}{\sqrt{2}}(\rho^+ + \rho^-), \\ V_2 &= \frac{i}{\sqrt{2}}(\rho^+ - \rho^-), \\ V_3 &= \rho^0, \\ V_4 &= \frac{1}{\sqrt{2}}(K^{*+} + K^{*-}), \\ V_5 &= \frac{i}{\sqrt{2}}(K^{*+} - K^{*-}), \\ V_6 &= \frac{1}{\sqrt{2}}(K^{*0} + \bar{K}^{*0}), \\ V_7 &= \frac{i}{\sqrt{2}}(K^{*0} - \bar{K}^{*0}), \\ V_8 &= \phi \cos \theta_V + \omega \sin \theta_V, \\ V_0 &= -\phi \sin \theta_V + \omega \cos \theta_V, \end{aligned} \quad (10)$$

then we calculate in detail on the effective Lagrangian of  $J/\psi \rightarrow VP$  decays

$$\begin{aligned} P_1^a V_2^a &= P^1 V^1 + P^2 V^2 + P^3 V^3 + P^4 V^4 + P^5 V^5 \\ &\quad + P^6 V^6 + P^7 V^7 + P^8 V^8 \\ &= \pi^+ \rho^- + \pi^- \rho^+ + \pi^0 \rho^0 + K^+ K^{*-} \\ &\quad + K^- K^{*+} + K^0 \bar{K}^{*0} + \bar{K}^0 K^{*0} \\ &\quad + \eta \omega \cos \theta_P \sin \theta_V + \eta \phi \cos \theta_P \cos \theta_V \\ &\quad + \eta' \omega \sin \theta_P \sin \theta_V + \eta' \phi \sin \theta_P \cos \theta_V, \end{aligned} \quad (11)$$

$$S_1 S_2 = P^0 V^0 = -\eta \omega \sin \theta_P \cos \theta_V + \eta \phi \sin \theta_P \sin \theta_V \\ + \eta' \omega \cos \theta_P \cos \theta_V - \eta' \phi \cos \theta_P \sin \theta_V, \quad (12)$$

$$d_{abc} O_1^a O_2^b M_3^c = d_{ab8} P^a V^b = d_{118} P^1 V^1 + d_{228} P^2 V^2 \\ + d_{338} P^3 V^3 + d_{448} P^4 V^4 + d_{558} P^5 V^5 \\ + d_{668} P^6 V^6 + d_{778} P^7 V^7 + d_{888} P^8 V^8 \\ = \frac{1}{\sqrt{3}} [\pi^+ \rho^- + \pi^- \rho^+ + \pi^0 \rho^0] \\ + \frac{-1}{2\sqrt{3}} [K^+ K^{*-} + K^- K^{*+} + K^0 \bar{K}^{*0} \\ + \bar{K}^0 K^{*0}] + \frac{-1}{\sqrt{3}} [\eta \omega \cos \theta_P \sin \theta_V \\ + \eta \phi \cos \theta_P \cos \theta_V + \eta' \omega \sin \theta_P \sin \theta_V \\ + \eta' \phi \sin \theta_P \cos \theta_V], \quad (13)$$

$$O_1^a M_3^a S_2 = P^8 V^0 = \eta \omega \cos \theta_P \cos \theta_V - \eta \phi \cos \theta_P \sin \theta_V \\ + \eta' \omega \sin \theta_P \cos \theta_V - \eta' \phi \sin \theta_P \sin \theta_V, \quad (14)$$

$$O_2^a M_3^a S_1 = P^0 V^8 = -\eta \omega \sin \theta_P \sin \theta_V - \eta \phi \sin \theta_P \cos \theta_V \\ + \eta' \omega \cos \theta_P \sin \theta_V + \eta' \phi \cos \theta_P \cos \theta_V, \quad (15)$$

$$d_{ab3} P^a V^b = d_{383} P^3 V^8 + d_{833} P^8 V^3 + d_{443} P^4 V^4 \\ + d_{553} P^5 V^5 + d_{663} P^6 V^6 + d_{773} P^7 V^7 \\ = \pi^0 \omega \sqrt{\frac{1}{3}} \sin \theta_V + \pi^0 \phi \sqrt{\frac{1}{3}} \cos \theta_V \\ + \eta \rho^0 \sqrt{\frac{1}{3}} \cos \theta_P + \eta' \rho^0 \sqrt{\frac{1}{3}} \sin \theta_P \\ + \frac{1}{2} K^+ K^{*-} + \frac{1}{2} K^- K^{*+} \\ - \frac{1}{2} K^0 \bar{K}^{*0} - \frac{1}{2} \bar{K}^0 K^{*0}, \quad (16)$$

$$d_{abc} O_1^a O_2^b E_3^c = d_{ab3} P^a V^b + \frac{1}{\sqrt{3}} d_{ab8} P^a V^b, \quad (17)$$

$$O_1^a E_3^a S_2 = P^3 V^0 + \frac{1}{\sqrt{3}} P^8 V^0, \quad (18)$$

$$P^3 V^0 = \pi^0 (\omega \cos \theta_V - \phi \sin \theta_V) \\ = \sqrt{\frac{2}{3}} \pi^0 \omega - \sqrt{\frac{1}{3}} \pi^0 \phi, \quad (19)$$

$$O_2^a E_3^a S_1 = V^3 P^0 + \frac{1}{\sqrt{3}} V^8 P^0, \quad (20)$$

$$V^3 P^0 = \rho^0 (-\eta \sin \theta_P + \eta' \cos \theta_P) \\ = -\eta \rho^0 \sin \theta_P + \eta' \rho^0 \cos \theta_P, \quad (21)$$

therefore we obtain the coupling constants and their corresponding strengths of  $J/\psi \rightarrow PV$  decays given in Table 1. It should be noted that our results are consistent with Ref. [2], but we distinguish in detail the different parameters of mass effect and electromagnetic effect.

The amplitude of  $J/\psi \rightarrow VP$  decays is

$$\mathcal{M} = \frac{g_{\psi VP}}{m_\psi} \varepsilon_{\mu\nu\rho\sigma} p_\psi^\mu \varepsilon_\psi^\nu p_V^\rho \varepsilon_V^{*\sigma}, \quad (22)$$

where  $g_{\psi VP}$  is the coupling constant and its various expressions are tabulated in Table 1, so we have

$$|\overline{\mathcal{M}}|^2 = \frac{1}{2s_\psi + 1} \sum_{s_\psi} \sum_{s_V} \mathcal{M} \mathcal{M}^\dagger \\ = \frac{2|g_{\psi VP}|^2}{3m_\psi^2} \left\{ \left( \frac{m_\psi^2 + m_V^2 - m_P^2}{2} \right)^2 - m_\psi^2 m_V^2 \right\} \\ = \frac{2}{3} |g_{\psi VP}|^2 p^2, \quad (23)$$

in which  $\vec{p}$  is the momenta of vector meson in the c.m. frame, and can be denoted as

$$p = |\vec{p}| = \frac{\sqrt{[m_\psi^2 - (m_V + m_P)^2][m_\psi^2 - (m_V - m_P)^2]}}{2m_\psi}. \quad (24)$$

The differential decay rate for  $J/\psi \rightarrow VP$  can be written as

$$d\Gamma = \frac{1}{32\pi^2} \frac{p}{m_\psi^2} |\overline{\mathcal{M}}|^2 d\Omega, \quad (25)$$

so we have the total decay width

$$\Gamma = \int \frac{1}{32\pi^2} \frac{p}{m_\psi^2} |\overline{\mathcal{M}}|^2 d\Omega = \frac{1}{3} \frac{p^3}{m_\psi^2} \frac{|g_{\psi VP}|^2}{4\pi}. \quad (26)$$

### III. NUMERICAL ANALYSIS AND DISCUSSIONS

In Table 1, we give out the free parameters involved in the decays  $J/\psi \rightarrow PV$ . In these parameters,  $g_M^{88}$ ,  $g_M^{81}$  and  $g_M^{18}$  denote the  $SU(3)$  mass breaking terms induced by the different quark mass,  $g_E^{88}$ ,  $g_E^{81}$  and  $g_E^{18}$  denote the  $SU(3)$  electromagnetic terms induced by the different quark electric charge. There is a phase angle, noted by  $\delta_E$ , between electromagnetic and strong interaction. We can take the coupling constants of mass effect as real, and denote the coupling constants of electromagnetic effect with  $g_E^i = |g_E^i| e^{i\delta_E}$ . Therefore there are eleven free parameters in the decays  $J/\psi \rightarrow PV$ , they are  $g_8$ ,  $g_1$ ,  $g_M^{88}$ ,  $g_M^{81}$ ,  $g_M^{18}$ ,  $|g_E^{88}|$ ,  $|g_E^{81}|$ ,  $|g_E^{18}|$ ,  $\delta_E$ ,  $\theta_P$  and  $\theta_V$ . For the sake of simplicity, we shall take the following four cases to analyze and discuss the properties of these parameters:

Case I: Assuming  $g_M^{88} = g_M^{81} = g_M^{18} = g_M$ ,  $g_E^{88} = g_E^{81} = g_E^{18} = g_E$ , and taking the mixing between  $\phi$  and  $\omega$  regard

Table 1 The coupling constants and their corresponding strengths of  $J/\psi \rightarrow PV$ .

Decay modes $J/\psi \rightarrow PV$	Coupling constants and their corresponding strengths							
	$g_8$	$g_1$	$g_M^{88}$	$g_M^{81}$	$g_M^{18}$	$g_E^{88}$	$g_E^{81}$	$g_E^{18}$
$\pi^+ \rho^-$	1	0	$\frac{1}{\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$\pi^- \rho^+$	1	0	$\frac{1}{\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$\pi^0 \rho^0$	1	0	$\frac{1}{\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$K^+ K^{*-}$	1	0	$\frac{1}{2\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$K^- K^{*+}$	1	0	$\frac{1}{2\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$K^0 \bar{K}^{*0}$	1	0	$\frac{1}{2\sqrt{3}}$	0	0	$\frac{2}{3}$	0	0
$\bar{K}^0 K^{*0}$	1	0	$\frac{1}{2\sqrt{3}}$	0	0	$\frac{2}{3}$	0	0
$\pi^0 \omega$	0	0	0	0	0	$\sqrt{\frac{1}{3}} \sin \theta_V$	$\sqrt{\frac{2}{3}} \cos \theta_V$	0
$\pi^0 \phi$	0	0	0	0	0	$-\sqrt{\frac{1}{3}} \cos \theta_V$	$\sqrt{\frac{2}{3}} \sin \theta_V$	0
$\eta \rho^0$	0	0	0	0	0	$\sqrt{\frac{1}{3}} \cos \theta_P$	0	$-\sqrt{\frac{2}{3}} \sin \theta_P$
$\eta' \rho^0$	0	0	0	0	0	$\sqrt{\frac{1}{3}} \sin \theta_P$	0	$\sqrt{\frac{2}{3}} \cos \theta_P$
$\eta \omega$	$\cos \theta_P \sin \theta_V$	$-\sin \theta_P \cos \theta_V$	$\frac{1}{\sqrt{3}} \cos \theta_P \sin \theta_V$	$\sqrt{\frac{2}{3}} \cos \theta_P \cos \theta_V$	$-\sqrt{\frac{2}{3}} \sin \theta_P \cos \theta_V$	$\frac{1}{\sqrt{3}}$ 4th colu.	$\frac{1}{\sqrt{3}}$ 5th colu.	$\frac{1}{\sqrt{3}}$ 6th colu.
$\eta \phi$	$\cos \theta_P \cos \theta_V$	$\sin \theta_P \sin \theta_V$	$-\frac{1}{\sqrt{3}} \cos \theta_P \cos \theta_V$	$-\sqrt{\frac{2}{3}} \cos \theta_P \sin \theta_V$	$-\sqrt{\frac{2}{3}} \sin \theta_P \sin \theta_V$	$\frac{1}{\sqrt{3}}$ 4th colu.	$\frac{1}{\sqrt{3}}$ 5th colu.	$\frac{1}{\sqrt{3}}$ 6th colu.
$\eta' \omega$	$\sin \theta_P \sin \theta_V$	$\cos \theta_P \cos \theta_V$	$\frac{1}{\sqrt{3}} \sin \theta_P \sin \theta_V$	$\sqrt{\frac{2}{3}} \sin \theta_P \cos \theta_V$	$\sqrt{\frac{2}{3}} \cos \theta_P \cos \theta_V$	$\frac{1}{\sqrt{3}}$ 4th colu.	$\frac{1}{\sqrt{3}}$ 5th colu.	$\frac{1}{\sqrt{3}}$ 6th colu.
$\eta' \phi$	$\sin \theta_P \cos \theta_V$	$-\cos \theta_P \sin \theta_V$	$-\frac{1}{\sqrt{3}} \sin \theta_P \cos \theta_V$	$-\sqrt{\frac{2}{3}} \sin \theta_P \sin \theta_V$	$\sqrt{\frac{2}{3}} \cos \theta_P \sin \theta_V$	$\frac{1}{\sqrt{3}}$ 4th colu.	$\frac{1}{\sqrt{3}}$ 5th colu.	$\frac{1}{\sqrt{3}}$ 6th colu.

as idea mixing, i.e.,  $\theta_V \approx 35.3^\circ$ . In this case, there are only six parameters, they are  $g_8$ ,  $g_1$ ,  $g_M$ ,  $|g_E|$ ,  $\delta_E$  and  $\theta_P$ .

Case II: Assuming  $g_M^{88} = g_M^{81} = g_M^{18} = g_M$ ,  $g_E^{88} = g_E^{81} = g_E^{18} = g_E$ , but taking the mixing angle  $\theta_V$  regard as free. In this case, the parameters have seven which are  $g_8$ ,  $g_1$ ,  $g_M$ ,  $|g_E|$ ,  $\delta_E$ ,  $\theta_P$ ,  $\theta_V$  respectively.

Case III: Assuming the three parameters of mass effects aren't equal, and the three parameters of electromagnetic effects aren't equal, either, they are all free parameters, but taking  $\theta_V \approx 35.3^\circ$ , then we shall have 10 parameters:  $g_8$ ,  $g_1$ ,  $g_M^{88}$ ,  $g_M^{81}$ ,  $g_M^{18}$ ,  $|g_E^{88}|$ ,  $|g_E^{81}|$ ,  $|g_E^{18}|$ ,  $\delta_E$ , and  $\theta_P$ .

Case IV: Assuming the three parameters of mass effects aren't equal, and the three parameters of electromagnetic effects aren't equal, either, further taking the mixing angle  $\theta_V$  as free in the same, then all of the eleven parameters are free.

For the latter two cases, we have to request more experimental information to analyze them due to too many parameters.

The experimental results for the decays  $J/\psi \rightarrow PV$  are mainly from Mark-III [11], DM2 [12] and BES [13]-[14], which are shown in Table 2. The last column in this table is the latest world average in 2012 [1]. To clarify the results obtained from different data set, we divided them into three subsections to investigate the properties of the coupling constants of the decays  $J/\psi \rightarrow PV$ .

#### A. Analysis of $J/\psi \rightarrow VP$ from MarkIII, DM2 and PDG2012 data in Case I

First, we consider the simplest case, i.e., Case I. In Ref. [15], the pseudoscalar mixing in  $J/\psi$  and  $\psi(2S)$  de-

cays has been analyzed, which shows  $\eta$  favors only consist of light quarks and  $\eta'$  has a room for gluonium admixture. However, in this paper we shall only consider their quark content in order to analyze the properties of coupling constants from  $SU(3)$  breaking. The results of fit are given in Table 3. From this table, we can obtain the following results: (i) the coupling constant of the octet strong interaction,  $g_8$ , is about twice larger than that one of the singlet,  $g_1$ ; (ii) the  $SU(3)$  breaking coupling constant from electromagnetic effect is large, about the same order of  $g_8$  and  $g_1$ , however, the  $SU(3)$  breaking coupling constant from mass effect is rather small, about smaller one order than those of  $g_8$  and  $g_1$ , moreover its uncertainty is also very large; (iii) the phase angle between strong and electromagnetic interaction is about  $\frac{2}{5}\pi$ ; (iv) the mixing angle in  $\eta$  and  $\eta'$ ,  $\theta_P$ , is about  $-20^\circ$ , which is consistent with the reasonable range  $-20^\circ \sim -10^\circ$  [16]; and (v) compared with the results of MarkIII and DM2 data, the goodness of fit to PDG2012 data is very large.

#### B. Analysis of $J/\psi \rightarrow VP$ from MarkIII, DM2 and PDG2012 data in Case II

Next, we consider case II, i.e.,  $\omega - \phi$  mixing angle is left as a free parameter. The results of fit are listed in Table 4. From this table, we can see that, the results of this case is similarly those of Case I only, but the results of fit to MarkIII data,  $\theta_P = -15.2 \pm 2.93$  are obvious difference with those of Case I. The table also shows that the mixing in  $\omega$  and  $\phi$  is basically an idea mixing.

Table 2 Branching fractions of  $J/\psi \rightarrow VP$  from MarkIII, DM2, BES and PDG2012 ( $\times 10^{-3}$ ).

Decay modes	MarkIII	DM2	BES	PDG2012
$\rho\pi$	$14.2 \pm 0.1 \pm 1.9$	$13.2 \pm 2.0$	$21.8 \pm 0.05 \pm 2.01$	$16.9 \pm 1.5$
$\rho\eta$	$0.193 \pm 0.013 \pm 0.029$	$0.194 \pm 0.017 \pm 0.029$		$0.193 \pm 0.023$
$\rho\eta'$	$0.114 \pm 0.014 \pm 0.016$	$0.083 \pm 0.030 \pm 0.012$		$0.105 \pm 0.018$
$\phi\pi$	$< 0.0068$		$< 0.0064$	$< 0.0064$
$\phi\eta$	$0.661 \pm 0.045 \pm 0.078$	$0.64 \pm 0.04 \pm 0.11$	$0.898 \pm 0.024 \pm 0.089$	$0.75 \pm 0.08$
$\phi\eta'$	$0.308 \pm 0.034 \pm 0.036$	$0.41 \pm 0.03 \pm 0.08$	$0.546 \pm 0.031 \pm 0.056$	$0.40 \pm 0.07$
$\omega\pi$	$0.482 \pm 0.019 \pm 0.064$	$0.360 \pm 0.028 \pm 0.054$	$0.538 \pm 0.012 \pm 0.065$	$0.45 \pm 0.05$
$\omega\eta$	$1.71 \pm 0.08 \pm 0.20$	$1.43 \pm 0.10 \pm 0.21$	$2.352 \pm 0.273$	$1.74 \pm 0.20$
$\omega\eta'$	$0.166 \pm 0.017 \pm 0.019$	$0.18^{+0.10}_{-0.08} \pm 0.03$	$0.226 \pm 0.043$	$0.182 \pm 0.021$
$K^{*-}K^+ + \text{c.c.}$	$5.26 \pm 0.13 \pm 0.53$	$4.57 \pm 0.17 \pm 0.70$		$5.12 \pm 0.30$
$K^{*0}\bar{K}^0 + \text{c.c.}$	$4.33 \pm 0.12 \pm 0.45$	$3.96 \pm 0.15 \pm 0.60$		$4.39 \pm 0.31$

Table 3 Results of fit to MarkIII, DM2 and PDG2012 data in Case I.

Parameter	MarkIII	DM2	PDG2012
$g_8 (\times 10^{-3})$	$5.89 \pm 0.18$	$5.57 \pm 0.22$	$5.98 \pm 0.13$
$g_1 (\times 10^{-3})$	$2.72 \pm 0.20$	$3.12 \pm 0.40$	$2.77 \pm 0.26$
$g_M (\times 10^{-4})$	$8.08 \pm 3.21$	$5.84 \pm 4.22$	$9.44 \pm 2.72$
$ g_E  (\times 10^{-3})$	$2.11 \pm 0.10$	$1.95 \pm 0.11$	$2.06 \pm 0.08$
$\delta_E$	$71.4 \pm 11.5$	$74.9 \pm 17.0$	$75.8 \pm 6.87$
$\theta_P$	$-19.5 \pm 1.52$	$-20.0 \pm 0.68$	$-19.0 \pm 1.59$
$\chi^2/d.o.f$	$7.19/4$	$3.63/4$	$19.4/4$

### C. Analysis of the branching ratios of $J/\psi \rightarrow VP$ in Case III and IV

Finally, we consider the latter two cases. There are 10 and 11 free parameters in Case III and IV, respectively. Because each of the experimental results are not sufficient do a reasonable fit for 10 or 11 free parameters in these two cases, we try to figure out these free parameters by combining the data from the different collaborations. Such as for Case III, the results of fit from MarkIII + BES data can be written as

$$\begin{aligned}
g_8 &= (6.57 \pm 0.16) \times 10^{-3}, \\
g_1 &= (3.14 \pm 0.15) \times 10^{-3}, \\
g_M^{88} &= (18.1 \pm 4.01) \times 10^{-4}, \\
g_M^{81} &= (5.78 \pm 4.20) \times 10^{-4}, \\
g_M^{18} &= (3.23 \pm 0.21) \times 10^{-4}, \\
|g_E^{88}| &= (1.73 \pm 0.22) \times 10^{-3}, \\
|g_E^{81}| &= (2.74 \pm 0.20) \times 10^{-3}, \\
|g_E^{18}| &= (2.18 \pm 0.15) \times 10^{-3}, \\
\delta_E &= 69.5 \pm 12.6, \quad \theta_P = -19.0 \pm 3.44. \quad (27)
\end{aligned}$$

we can see that, (i) the coupling constants of strong interaction,  $g_8$  and  $g_1$ , are basically consistent with those of Case I and Case II; (ii) the difference of three coupling constants of electromagnetic effect isn't obvious, but three coupling constants of mass effect have rather large difference, moreover their uncertainties are rather large; and (iii) the goodness of fit in this case is very large, it is a possible reason that we deal together with the data from the different collaborations.

For Case IV, the results of fit from MarkIII and BES

data are

$$\begin{aligned}
g_8 &= (6.57 \pm 0.16) \times 10^{-3}, \\
g_1 &= (3.05 \pm 0.15) \times 10^{-3}, \\
g_M^{88} &= (18.7 \pm 3.30) \times 10^{-4}, \\
g_M^{81} &= (11.0 \pm 1.47) \times 10^{-4}, \\
g_M^{18} &= (7.00 \pm 0.68) \times 10^{-4}, \\
|g_E^{88}| &= (1.81 \pm 0.22) \times 10^{-3}, \\
|g_E^{81}| &= (2.70 \pm 0.20) \times 10^{-3}, \\
|g_E^{18}| &= (2.14 \pm 0.15) \times 10^{-3}, \\
\delta_E &= 78.0 \pm 2.56, \quad \theta_P = -18.0 \pm 0.53, \\
\theta_V &= 35.9 \pm 1.47, \quad \chi^2/d.o.f = 36.1/5. \quad (28)
\end{aligned}$$

This case is similar to Case III, the difference of three coupling constants of electromagnetic effect isn't still obvious, three coupling constants of mass effect have rather large difference, and at the same time, the phase angle between strong and electromagnetic interaction is slightly large.

The results of fit from DM2+BES combination and MarkIII + DM2 combination, are similar to those of MarkIII + BES. Their numerical results aren't individually given out.

## IV. CONCLUSIONS

Basing on the general phenomenological model, we have studied the properties of the coupling constants of the decays  $J/\psi \rightarrow VP$ . Considering the experimental data of MarkIII, DM2 and BES collaborations and the

Table 4 Results of fit to MarkIII, DM2 and PDG2012 data in Case II.

Parameter	MarkIII	DM2	PDG2012
$g_8 (\times 10^{-3})$	$5.87 \pm 0.18$	$5.57 \pm 0.22$	$5.99 \pm 0.13$
$g_1 (\times 10^{-3})$	$2.84 \pm 0.21$	$3.12 \pm 0.42$	$2.84 \pm 0.29$
$g_M (\times 10^{-4})$	$5.75 \pm 3.40$	$5.88 \pm 4.84$	$8.42 \pm 3.32$
$ g_E  (\times 10^{-3})$	$2.15 \pm 0.10$	$1.94 \pm 0.12$	$2.07 \pm 0.08$
$\delta_E$	$71.7 \pm 11.1$	$74.9 \pm 16.9$	$76.0 \pm 6.81$
$\theta_P$	$-15.2 \pm 2.93$	$-20.1 \pm 5.06$	$-17.8 \pm 2.68$
$\theta_V$	$41.7 \pm 3.65$	$35.4 \pm 7.27$	$37.5 \pm 3.42$
$\chi^2/d.o.f$	4.95/3	3.63/3	19.2/3

world average in 2012 for  $J/\psi \rightarrow VP$  decays, we can find that (i) the octet coupling constant of strong interaction  $g_8$  is about twice larger than that of the singlet coupling constant  $g_1$ ; (ii) the electromagnetic breaking terms  $g_E^i$  are larger, about the same order of  $g_8$  and  $g_1$ , the difference of three coupling constants  $g_E^i$  isn't obviously, then it is appropriate that we take them regard as equal in Case I and II; (iii) the  $SU(3)$  breaking coupling constant from mass effect is rather small, about one order smaller than those of  $g_8$  and  $g_1$ . Moreover its uncertainty is quite large. It is a very strong assumption that we take them regard as equal in Case I and II; (iv) the phase angle between strong and electromagnetic interaction is about the range of  $70^\circ \sim 80^\circ$ ; (v) the mixing angle in  $\eta$  and  $\eta'$ ,  $\theta_P$ , is about the range of  $-15^\circ \sim -20^\circ$ , which is consistent with the reasonable range usually considered; (vi) the

mixing angle  $\theta_V$  is about the rang of  $35^\circ \sim 37^\circ$ , it means the  $\phi - \omega$  mixing is basically an idea mixing. Therefore in the breaking of  $SU(3)$  favor symmetry, the electromagnetic effects are large, moreover the mass effects are comparatively small, which affords useful information to comprehend the breaking of  $SU(3)$  flavor symmetry.

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